

# Confidence Intervals for Proportions

Lecture 28  
Sections 9.4

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# Outline

- 1 The Seven Steps of Hypothesis Testing
- 2 Coin-Tossing Example
- 3 Hypothesis Testing on the TI-83
- 4 Introduction to Estimation
- 5 The Target Analogy
- 6 The Confidence Interval
- 7 Example
- 8 Assignment

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# The Steps of Testing a Hypothesis

## $p$ -Value Approach

The seven steps of hypothesis testing.

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- 4 Compute the value of the test statistic.

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- 5 Compute the  $p$ -value.
- 6 Make a decision.

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- 3 State the formula for the test statistic.
- 4 Compute the value of the test statistic.
- 5 Compute the  $p$ -value.
- 6 Make a decision.
- 7 State the conclusion.

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# The Coin-Tossing Example

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- It had a 51% chance of landing heads.
- Why did the test show that it was fair?
- Or did it?

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- Altogether, I got 5148 heads out of 10,000 tosses.

# The Coin-Tossing Example

- I decided to toss the coin 9000 more times, for a total of 10,000 tosses.
- Out of those 9000 tosses, I got 4623 heads.
- Altogether, I got 5148 heads out of 10,000 tosses.
- Now run the test.

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# Hypothesis Testing on the TI-83

## TI-83 Hypothesis Testing for $p$

- Press `STAT`.
- Select the `TESTS` menu.
- Select `1-PropZTest . . .`
- Press `ENTER`. A window appears.
- Enter the value of  $p_0$ .
- Press `ENTER` and the down arrow.
- Enter the numerator  $x$  of  $\hat{p}$ .
- Press `ENTER` and the down arrow.
- Enter the sample size  $n$ .
- Press `ENTER` and the down arrow.

# Hypothesis Testing on the TI-83

## TI-83 Hypothesis Testing for $p$

- Select the type of alternative hypothesis.
- Press the down arrow.
- Select `Calculate`.
- Press `ENTER`.

# Hypothesis Testing on the TI-83

## TI-83 Hypothesis Testing for $p$

- The following appear in the display.
  - The title `1-PropZTest`.
  - The alternative hypothesis.
  - The value of the test statistic  $Z$ .
  - The  $p$ -value.
  - The value of  $\hat{p}$ .
  - The sample size  $n$ .

# Hypothesis Testing on the TI-83

- Work the coin-tossing example using the TI-83.

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# Point Estimates

## Definition (Point estimate)

A **point estimate** is a single number that estimates the value of the parameter.

- $\bar{x}$  is a point estimator for  $\mu$ .
- $\hat{p}$  is a point estimator for  $p$ .
- $s^2$  is a point estimator for  $\sigma^2$ .
- $s$  is a point estimator for  $\sigma$ .

# Point Estimates

- The problem with point estimates is that we have no idea how close we can expect them to be to the parameter.
- That is, they provide no idea of how large the error may be.

# Interval Estimates

## Definition (Interval estimate)

An **interval estimate** is an interval of numbers that has a specified probability of containing the value of the parameter.

## Definition (Margin of error)

The **margin of error** of an interval estimate is half the width of the interval.

- An interval estimate is more informative than a point estimate.

# Interval Estimates

- $\bar{x} \pm$  (margin of error) is an interval estimate for  $\mu$ .
- $\hat{p} \pm$  (margin of error) is an interval estimate for  $p$ .
- $s^2 \pm$  (margin of error) is an interval estimate for  $\sigma^2$ .
- $s \pm$  (margin of error) is an interval estimate for  $\sigma$ .

# Interval Estimates

## Question

How do we find the margin of error?

- It depends on...

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  - Maybe the sample size.

## Question

How do we find the margin of error?

- It depends on...
  - The parameter involved.
  - The level of confidence desired.
  - Maybe the sample size.
  - Maybe even some other stuff.

# Confidence Levels

## Definition (Confidence level)

The **confidence level** of an interval estimate is the probability that an interval calculated in this manner will contain the parameter.

- If the confidence level is 95%, then the interval estimate is called a **95% confidence interval**.
- Using the point estimate as the center of the interval estimate, we are 95% sure that the value of the parameter is within the margin of error from the point estimate.
- The correct interpretation is that of all intervals calculated in this manner, 95% of them will contain the parameter and the other 5% will not.

# 95% Confidence Intervals

- Given the sample size  $n$ , the sampling distribution of  $\hat{p}$  is normal with mean  $p$  and standard deviation

$$\sqrt{\frac{p(1-p)}{n}}.$$

- Therefore, 95% of the time,  $\hat{p}$  will lie within  $1.96\sqrt{\frac{p(1-p)}{n}}$  of  $p$ .

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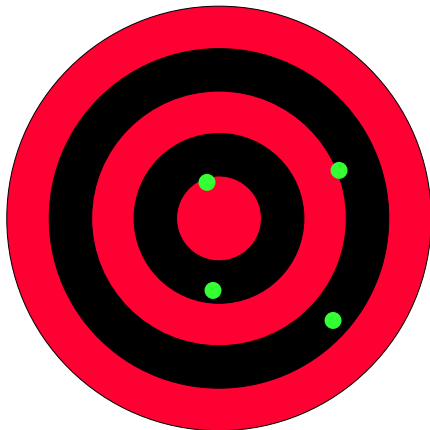
# The Target Analogy

- Consider an analogy of a shooter shooting at a target.
- Suppose a shooter hits within 5 rings (5 inches) of the bull's eye 95% of the time.
- Then each individual shot has a 95% chance of hitting within 5 inches.

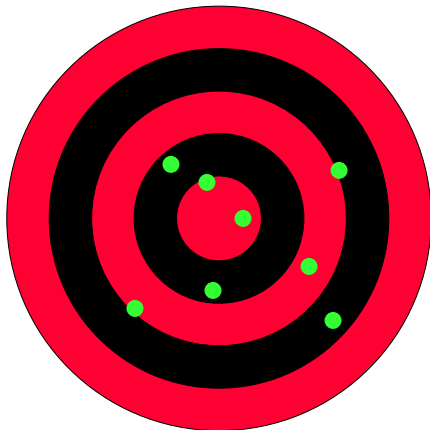
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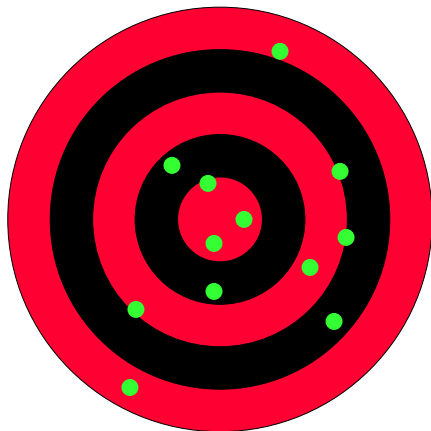
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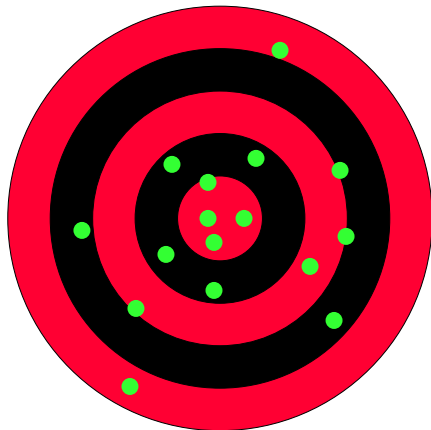
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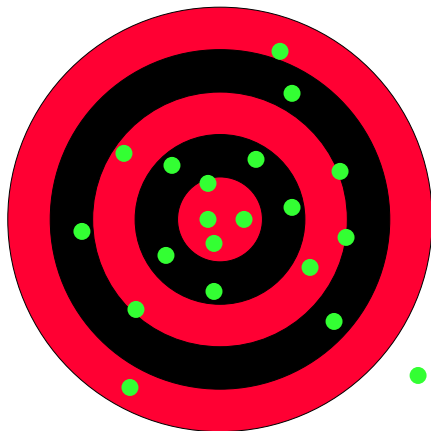
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# The Target Analogy

- On any given shot, the shooter can be 95% sure that he hit the target.

# The Target Analogy

- Now suppose we are shown where the shot hit, but we are not shown where the bull's eye is.
- What is the probability that the bull's eye is within 5 inches of that shot?

# The Target Analogy



# The Target Analogy



Where is the bull's eye?

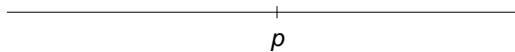
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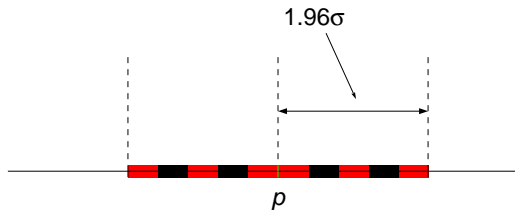
# The Confidence Interval

- In a similar way, 95% of the sample proportions  $\hat{p}$  should lie within 1.96 standard deviations ( $\sigma_{\hat{p}}$ ) of the parameter  $p$ .

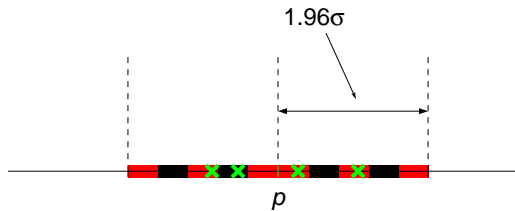
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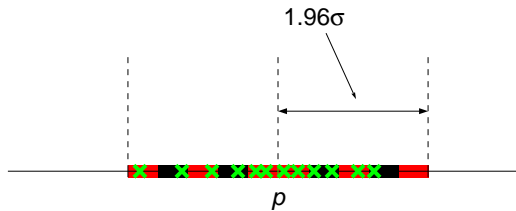


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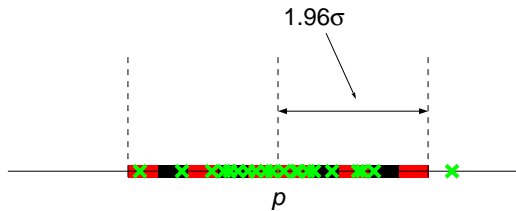


# The Confidence Interval





# The Confidence Interval



# The Confidence Interval

- Therefore, if we compute a single  $\hat{p}$ , then there is a 95% chance that it lies within a distance  $1.96\sigma_{\hat{p}}$  of  $p$ .

# The Confidence Interval



# The Confidence Interval

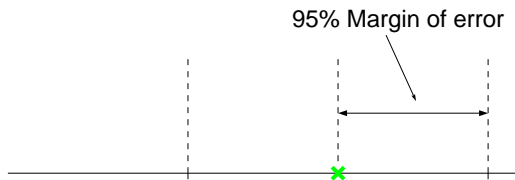


# The Confidence Interval



Where is  $p$ ?

# The Confidence Interval



# Approximate 95% Confidence Intervals

- Thus, the 95% confidence interval *would* be

$$\hat{p} \pm 1.96\sigma_{\hat{p}},$$

except...

# Approximate 95% Confidence Intervals

- The formula for  $\sigma_{\hat{p}}$  is

$$\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}},$$

which requires that we know  $p$ , the very thing we are trying to estimate!

# Approximate 95% Confidence Intervals

- The best we can do is to use  $\hat{p}$  in place of  $p$  to estimate  $\sigma_{\hat{p}}$ :

$$\hat{p} \pm 1.96 \sqrt{\frac{\hat{p}(1 - \hat{p})}{n}}.$$

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# Example

## Example (95% confidence interval for $p$ )

- In our first coin-tossing experiment, the coin landed heads 525 times out of 1000 tosses.
- Estimate, using a 95% confidence interval, the true probability of heads.

# Example

## Example (95% confidence interval for $p$ )

- The point estimate is

$$\hat{p} = \frac{525}{1000} = 0.525.$$

# Example

## Example (95% confidence interval for $p$ )

- The point estimate is

$$\hat{p} = \frac{525}{1000} = 0.525.$$

- The margin of error is

$$1.96\sqrt{\frac{\hat{p}(1 - \hat{p})}{n}} = 1.96\sqrt{\frac{(0.525)(0.475)}{1000}} = 0.03095.$$

# Example

## Example (95% confidence interval for $p$ )

- The point estimate is

$$\hat{p} = \frac{525}{1000} = 0.525.$$

- The margin of error is

$$1.96\sqrt{\frac{\hat{p}(1 - \hat{p})}{n}} = 1.96\sqrt{\frac{(0.525)(0.475)}{1000}} = 0.03095.$$

- The confidence interval is

$$\hat{p} \pm 1.96\sqrt{\frac{\hat{p}(1 - \hat{p})}{n}} = 0.525 \pm 0.03095.$$

# Example

## Example (95% confidence interval for $p$ )

- We are 95% confident that the point estimate of 0.525 is in error by no more than 0.03095.

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# Assignment

## Homework

- Read Section 9.4, pages 583 - 588.
- Let's Do It! 9.5.
- Exercises 19 - 25, 29, 30, page 595.
- Review exercises 39 - 44, 47 - 49, 51 - 54, page 607.